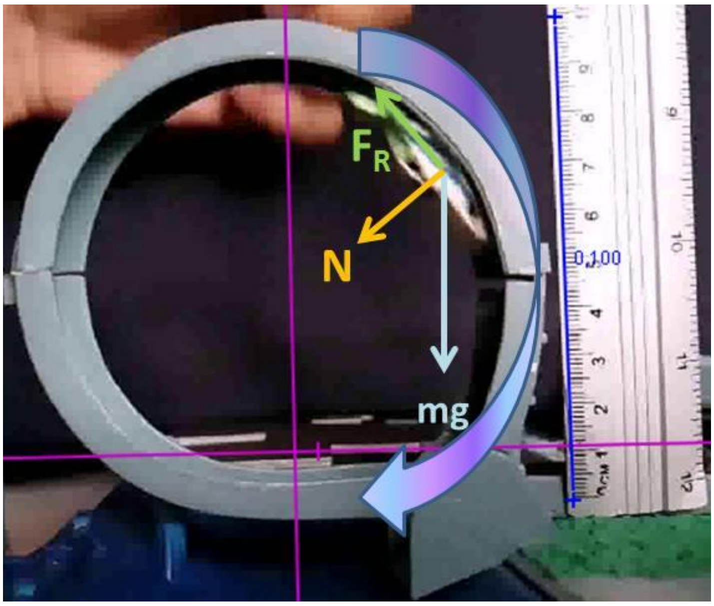
Car in a loop with friction



Summary

The motion of an object in a loop without friction is a classical example of normal acceleration problems with non-uniform circular motion and of energy conservation laws in Introductory Physics Courses. However, in real life toy car set-ups, friction becomes an important ingredient to determine whether a car will run beyond the top of the loop or not.

This experiment analyzes the motion of a toy car in a loop with a dynamical model that considers the action of three forces: gravity (mg), Normal Force (N) and Friction (F_R). A Cartesian coordinate system is used, with its origin at the initial point of the trajectory, at the bottom of the loop. The simulated trajectory obtained with Tracker Dynamic particle (Cartesian) model is compared with the experimental trajectory data from a slow motion video. The friction coefficient μ is adjusted empirically to $F_R = \mu$ N with μ=0.22 from the best match between model and experiment.

Equations of motion

Figure 1 shows a sketch of the velocity, acceleration and forces acting on a toy car moving around a loop. Newton's equation for a particle of mass m moving around a loop of radius R can be written as:

$$ma_t = mg\cos\theta - F_R$$
$$ma_n = mv^2/R = mg\sin\theta + N$$

where a_t and a_n are the tangential and normal components of the acceleration, v is the speed, N is the normal force and F_R is the friction force.

Thus, the normal and friction forces at each point can be calculated as

$$N = mv^2/R - mg\sin\theta$$
$$F_R = \mu N$$

and the acceleration vector in the fixed Cartesian coordinate system of figure 1, $\boldsymbol{a} = (a_x, a_y)$, can be calculated as

$$ma_x = -N\cos\theta - F_R\sin\theta$$
$$ma_y = -mg - N\sin\theta + F_R\cos\theta$$

with $\sin\theta = (y-R)/R$ and $\cos\theta = x/R$

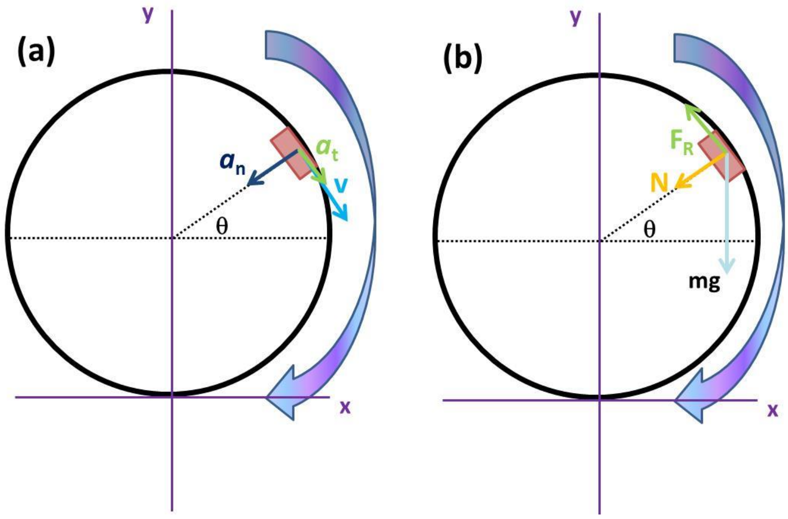


Figure 1: (a) Velocity and acceleration of a particle moving around a loop (b) Forces acting on a particle in a loop.

Results

Figure 2 is a comparison of experimental data extracted from a slow motion (240 fps) video of a car in a loop (red points) with a non friction dynamic model (green points).

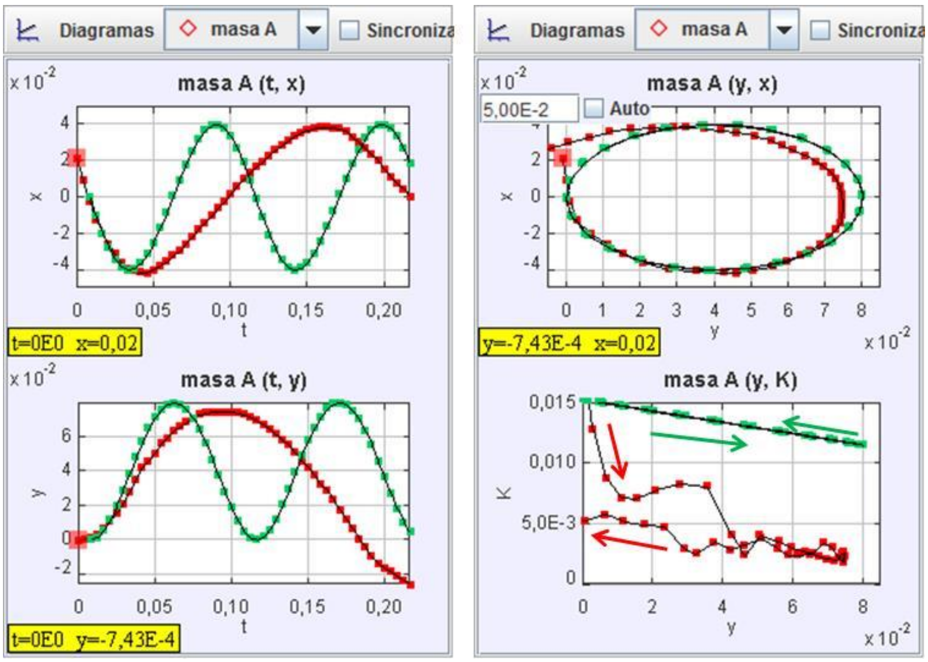


Figure 2: Comparison of experimental trajectory data of a car in a loop (red points) with an ideal conservative model, μ=0, (green points). Left panels: position vs. time x(t), y(t); right panels: circular trajectory y(x); Kinetic energy K vs. position. Arrows indicate the sense of motion.

Note that even though the trajectory (x,y) is a approximately a circle in both cases, the experimental motion is much slower (see x(t) and y(t) curves). The most important differences appear in the Kinetic energy plot. In the conservative model (green points), the kinetic energy follows a reversible curve and returns to its initial value when the car returns to its initial position. In the experimental data, the kinetic energy decreases by a factor of four along a single rotation within the loop.

Figure 3 shows the comparison between experimental data and a more realistic model including a friction force with μ=0.22.

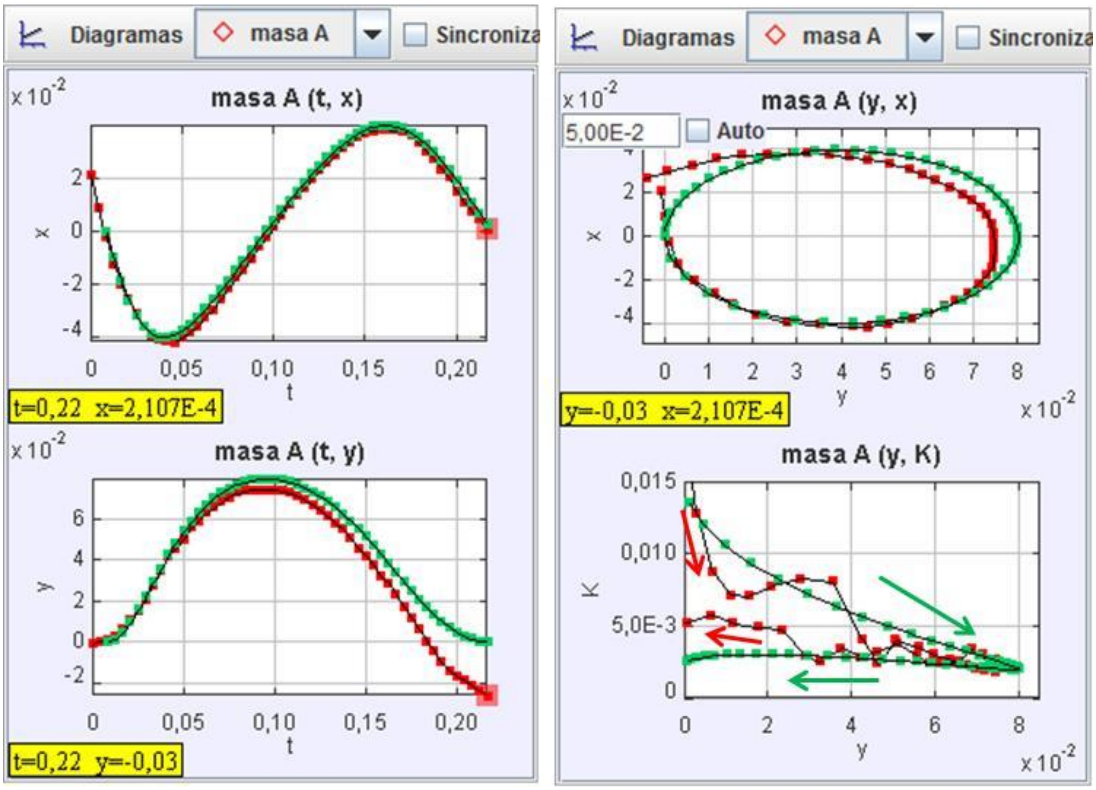


Figure 3: Comparison of experimental trajectory data of a car in a loop (red points) with a more realistic non conservative model, μ=0.22, (green points). Left panels: position vs. time x(t), y(t); right panels: circular trajectory y(x); Kinetic energy K vs. position. Arrows indicate the sense of motion.